

Tsung-Yu Tsai

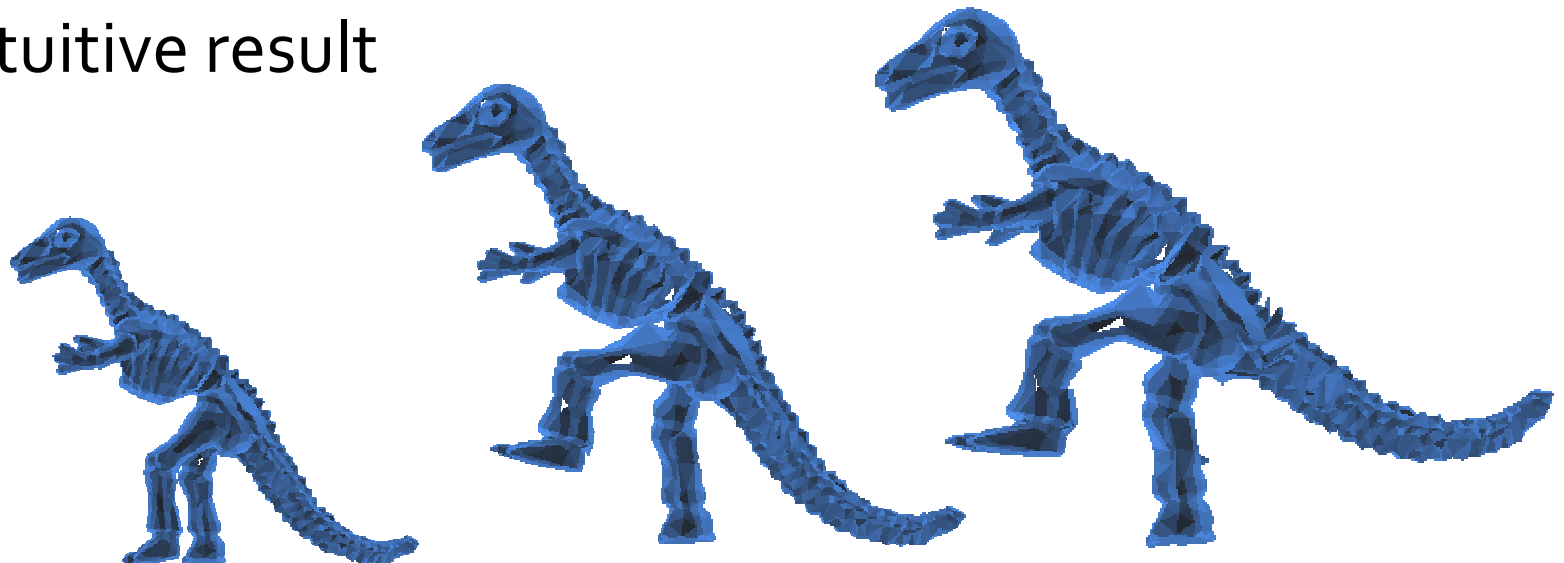
# Constraints-based 3D Model Deformation

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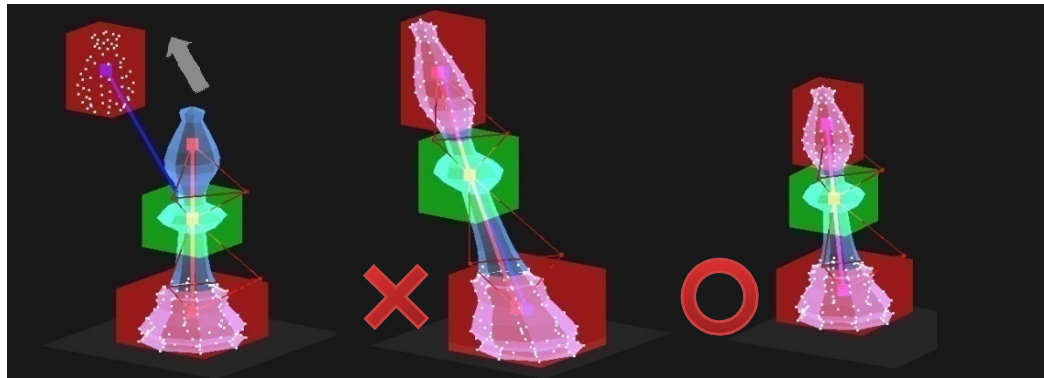
# Introduction

- We build a mesh deformation system
  - A user-friendly interface for easy manipulation
  - Detail preservation
  - Satisfying lots constraints
  - Intuitive result

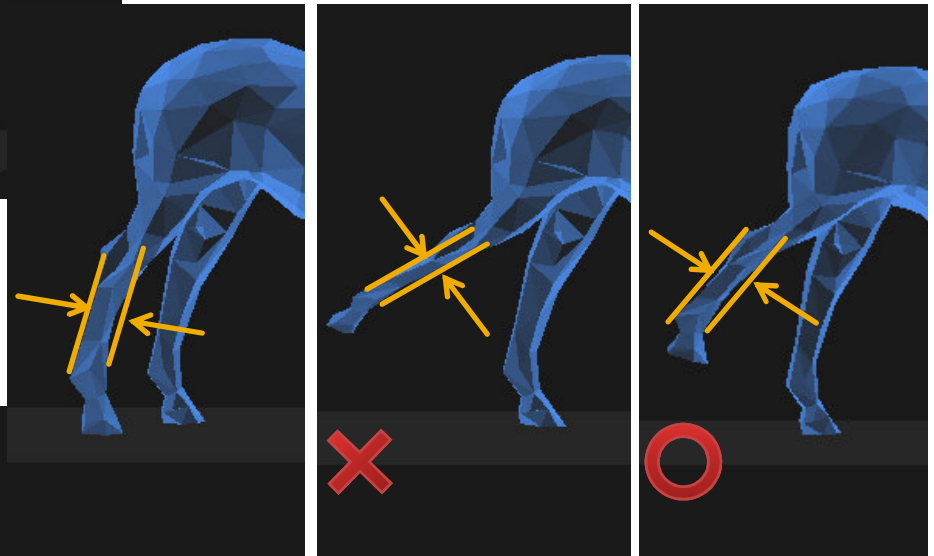


# Introduction

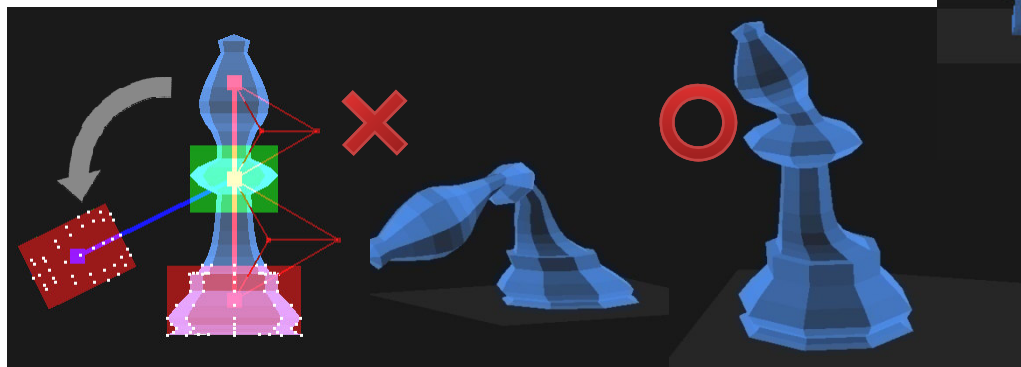
## Length constraint



## Rigidity constraint



## Joint angle constraint



# Outline

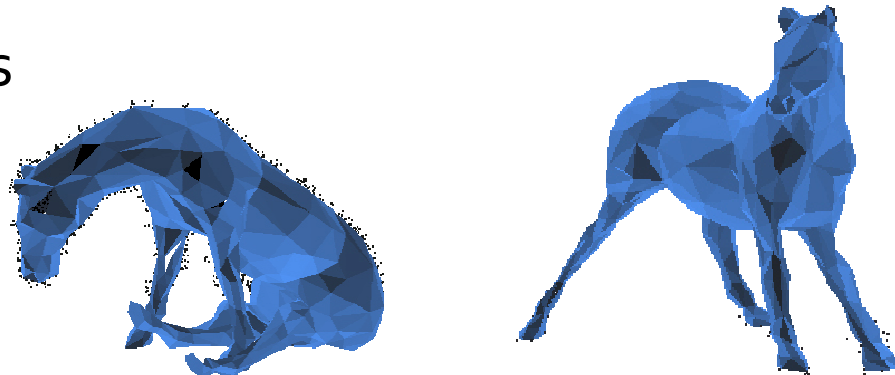
- Introduction
- Related Work
- System
- Results
- Conclusions and Future Work

# Outline

- Introduction
- **Related Work**
- System
- Results
- Conclusions and Future Work

# Related Work

- Mesh Deformation
  - Motivation : Mesh editing
    - Creating and modifying the shape of model
    - Modify global shape
      - Preserve local features and global continuity
    - Simple control mechanism
    - Intuitive results

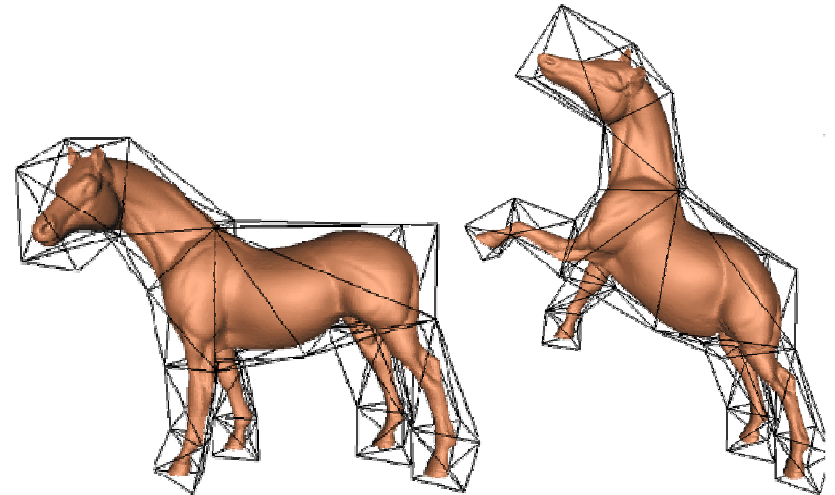


# Related Work

- Mesh deformation
  - Detail preservation based on local coordinates
  - **Laplacian surface editing** [*Sorkine et al. 2005*]
    - **Laplacian coordinates**
    - Cast mesh deformation as an energy minimization problem
  - The optimizations involved are often nonlinear and require Gauss-Newton iterations
    - Slow-converging
  - The limitation can be overcome through
    - **Linear solver (faster)** [*Lipman et al. 2004, Zhou et al. 2005*]

# Related Work

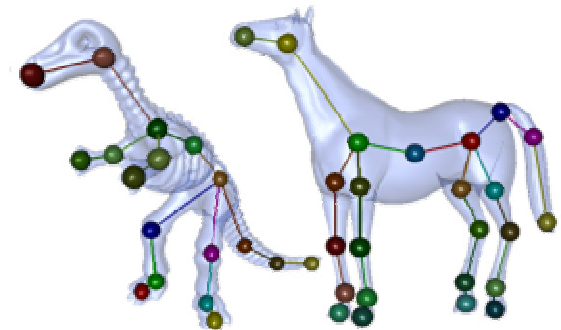
- **Mean value coordinates for closed triangular meshes** [*Ju et al. 2005*]
  - A coarser mesh embedding the mesh model
    - Interpolate values assigned to the vertices of a closed mesh
  - The disadvantage
    - Not convenient for user to control
- **Other manipulation**
  - Control handle
  - **Rigging**





# Related Work

- **Mesh Puppetry** [Zhou et al. 2007]
  - Direct manipulation & detail preservation
  - A set of high-level IK constraints (length, rigidity, joint limit, balance)
  - A cascading optimization procedure
- Our system
  - Easy manipulation & Rigging
  - Satisfying high-level constraints
  - Linear solver for deformation energy function

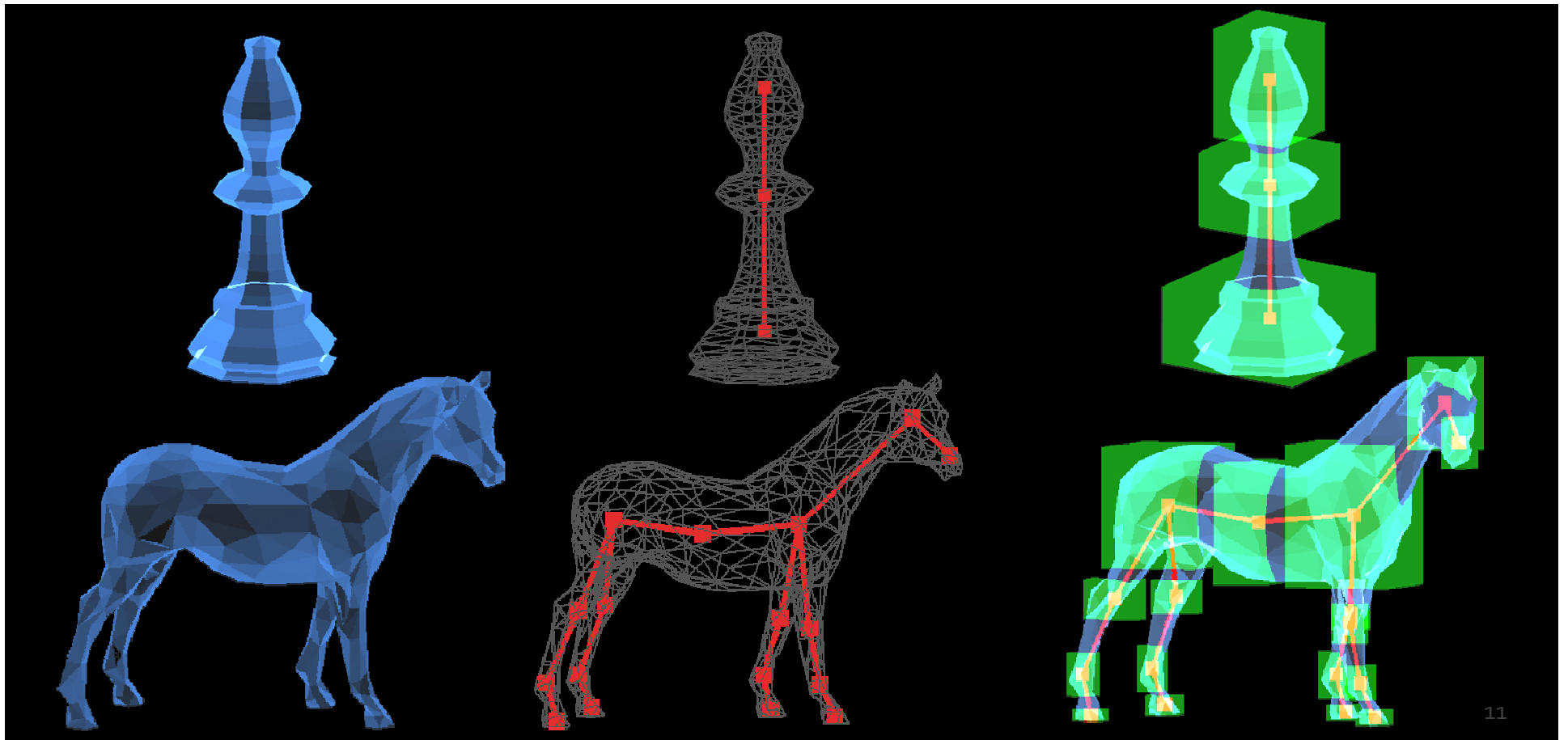


# Outline

- Introduction
- Related Work
- **System**
- Results
- Conclusions and Future Work

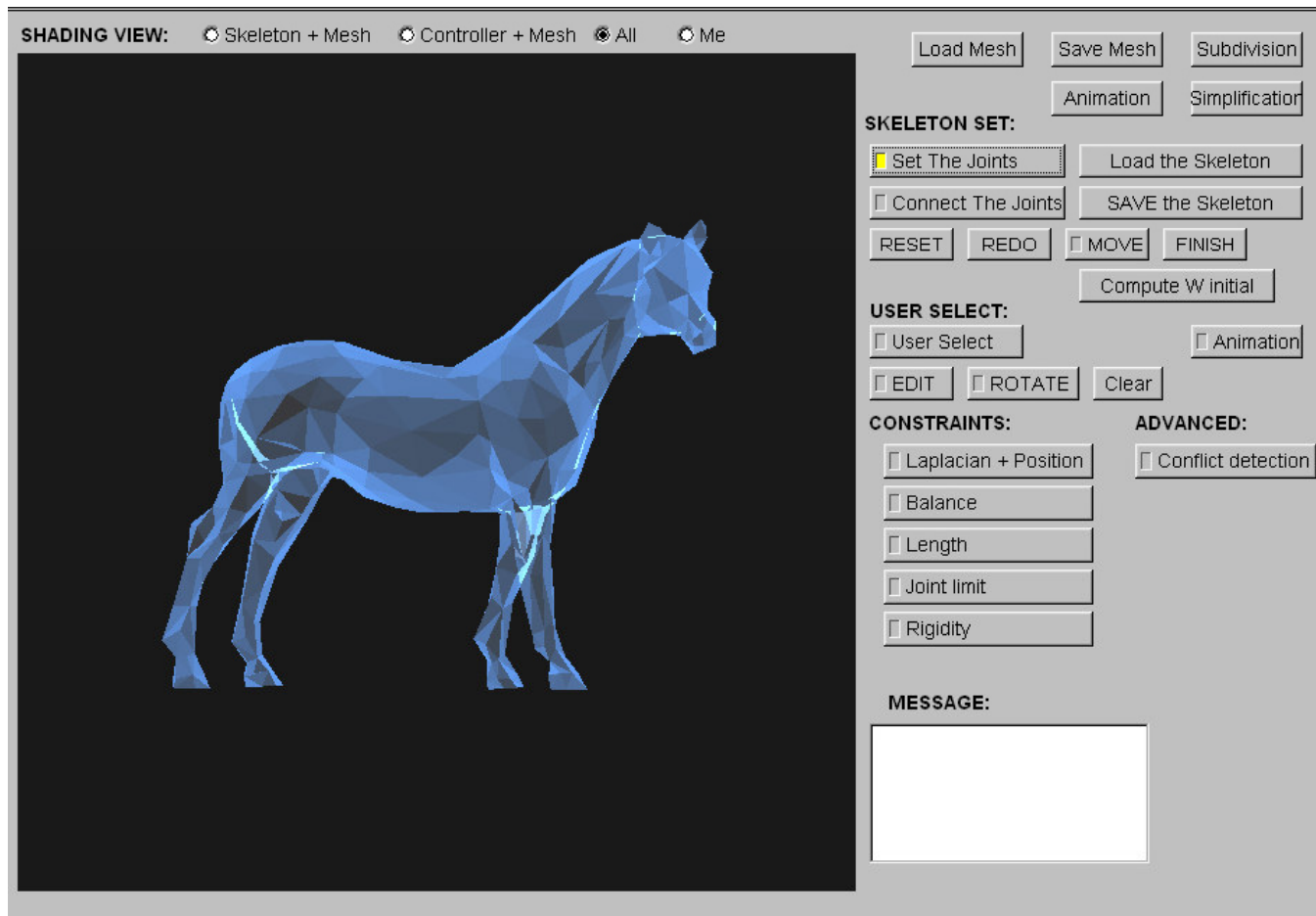
# System

- Input: Mesh + skeleton



# System

- How to build the skeleton ?



# System

- Steps of manipulation

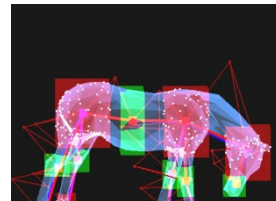
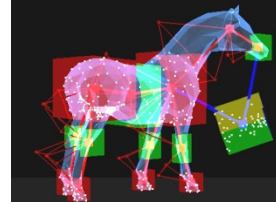
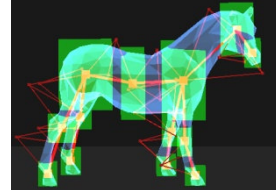
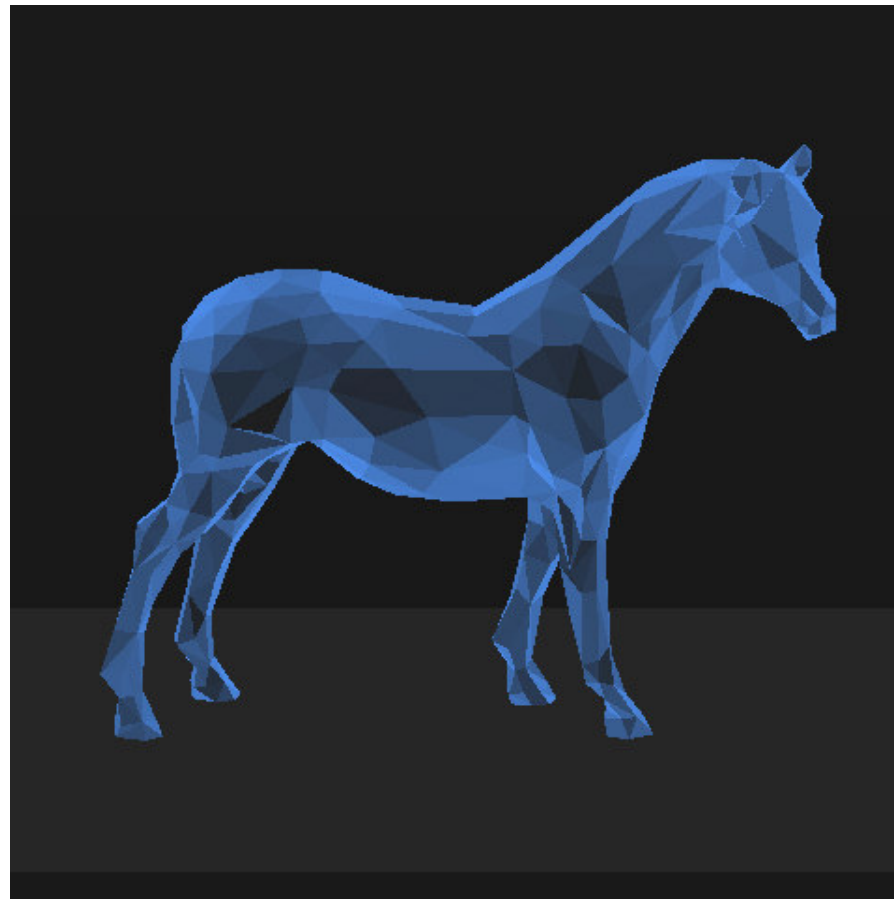
Load Model

Load skeleton

Move  
the selected joint

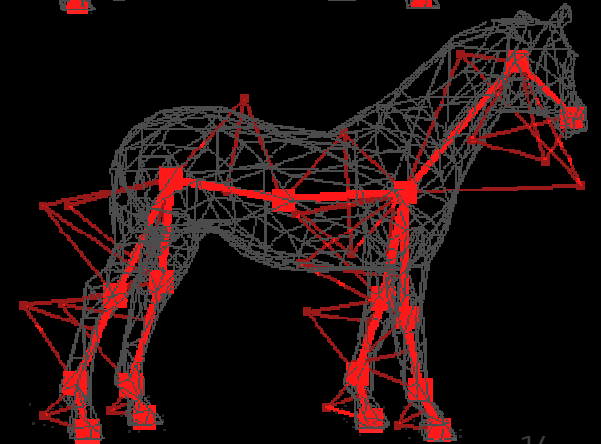
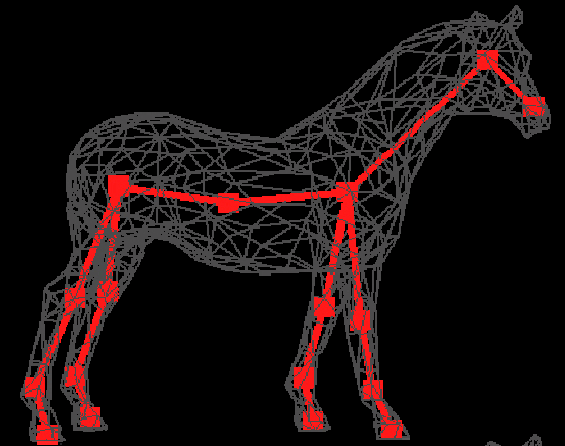
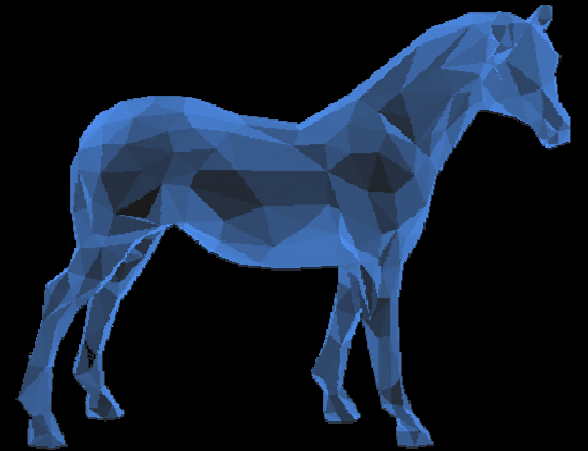
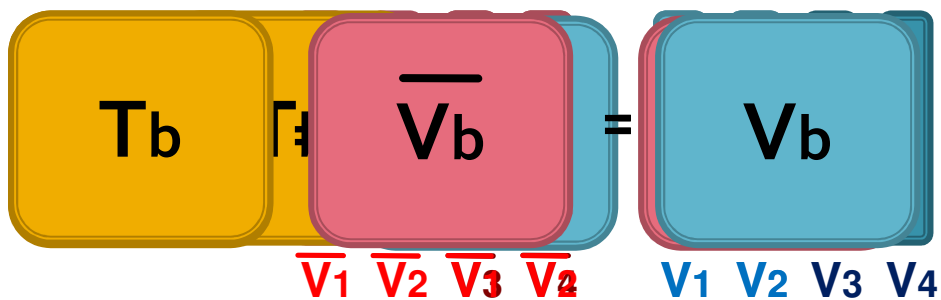
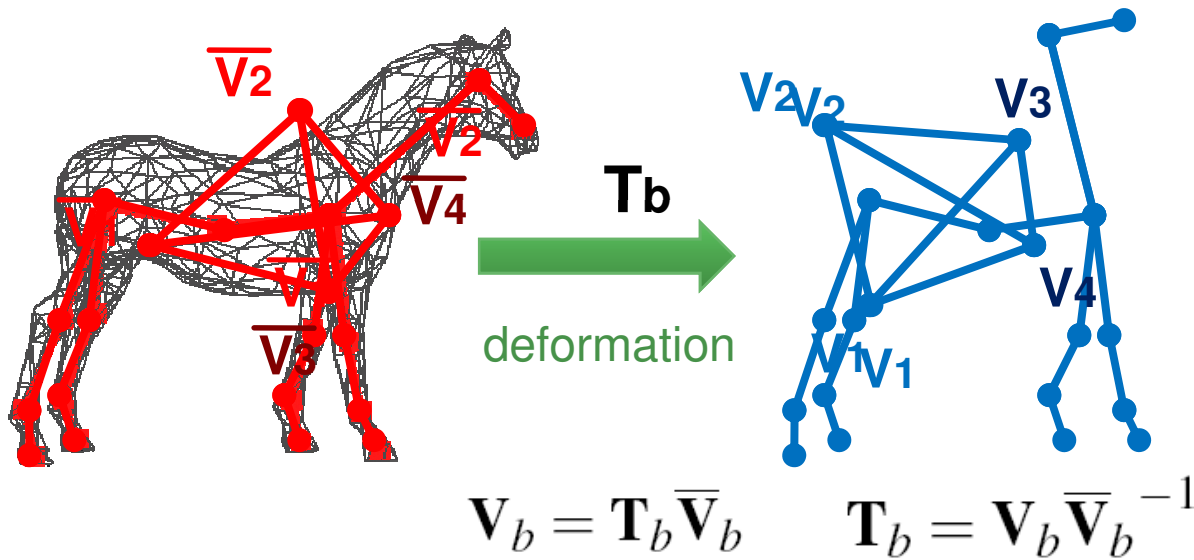
Deformation

Result



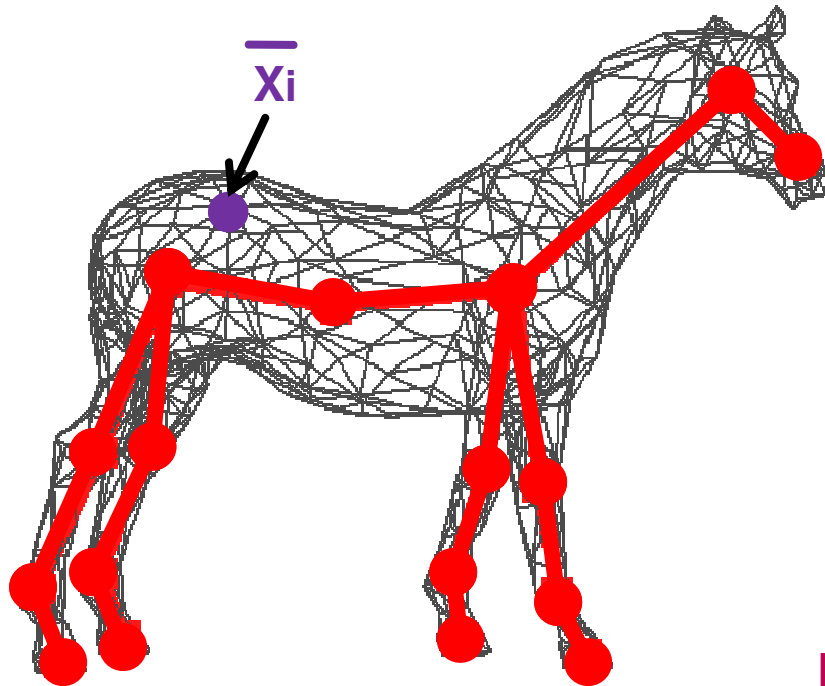
# System

- Tetrabone [Zhou et al. 2007]

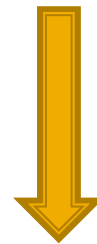


# System

- How to get the deformed mesh ?



$$\mathbf{x}_i = \mathbf{X} = \left( \prod_{b \in \text{bones}} \mathbf{T}_b \mathbf{W} \bar{\mathbf{X}}_b \right) \bar{\mathbf{x}}_i$$



$$\mathbf{T}_b = \mathbf{V}_b \bar{\mathbf{V}}_b^{-1}$$

$$\mathbf{x}_i = \mathbf{X} = \left( \prod_{b \in \text{bones}} \mathbf{V}_b \bar{\mathbf{V}}_b^{-1} \mathbf{W} \bar{\mathbf{X}}_b \right) \bar{\mathbf{x}}_i$$

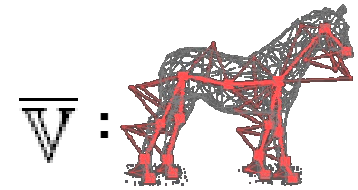
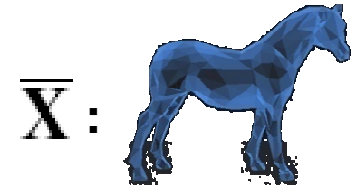
Deformed mesh model

# System

- The output of skinned mesh

$$\mathbf{X} = \mathbf{V} \mathbf{V}^{-1} \mathbf{W} \bar{\mathbf{X}}$$

We want to get them!!



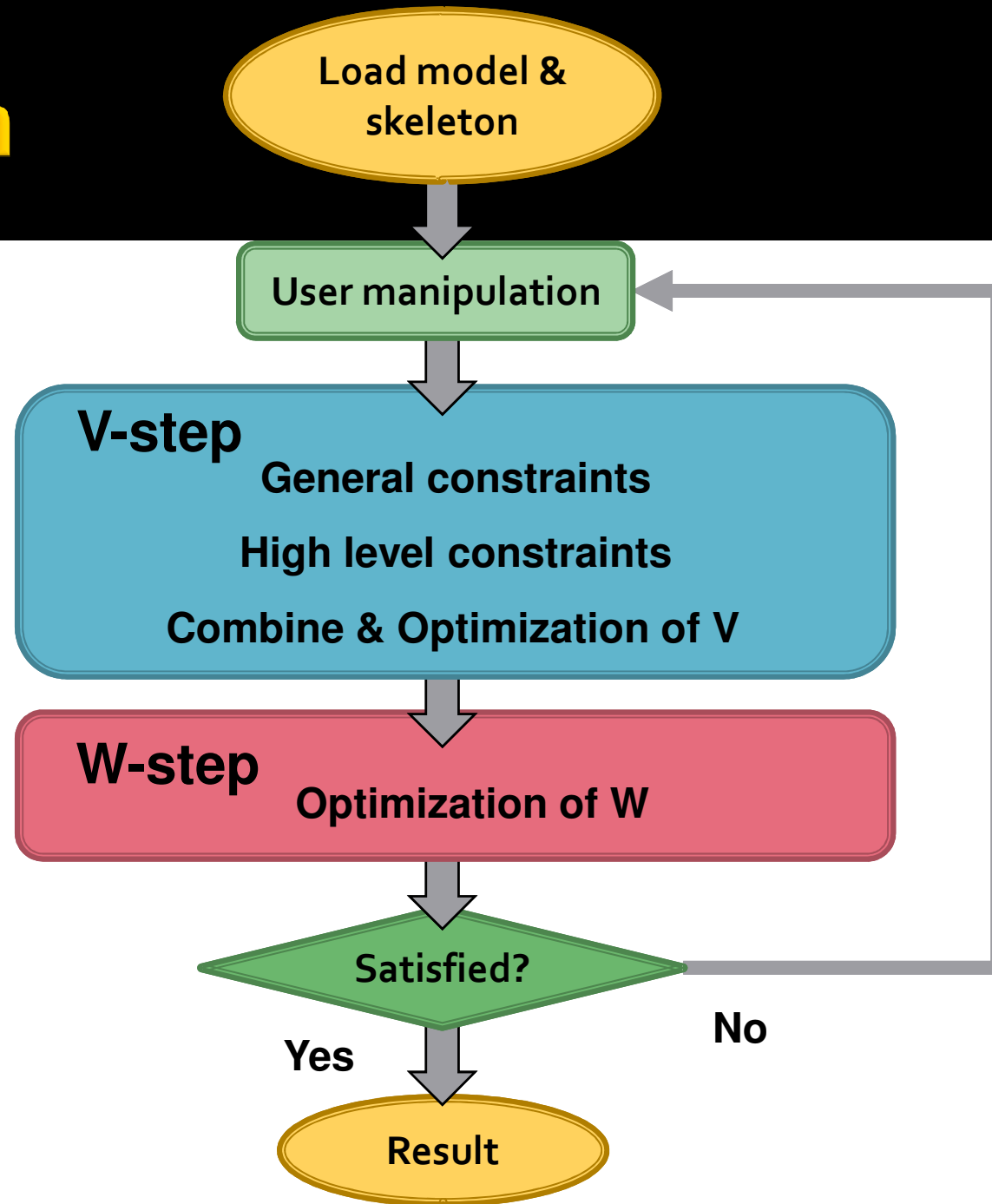
- We look up for a deformed mesh with vertex position  $\mathbf{X}$  (as a function of  $\mathbf{V}$  and  $\mathbf{W}$ )
  - Minimize the global deformation energy

$$\mathcal{M} = \underset{\mathbf{X} = \mathbf{V} \mathbf{V}^{-1} \mathbf{W} \bar{\mathbf{X}}}{\text{arg min}} \mathcal{E}(\mathbf{X})$$



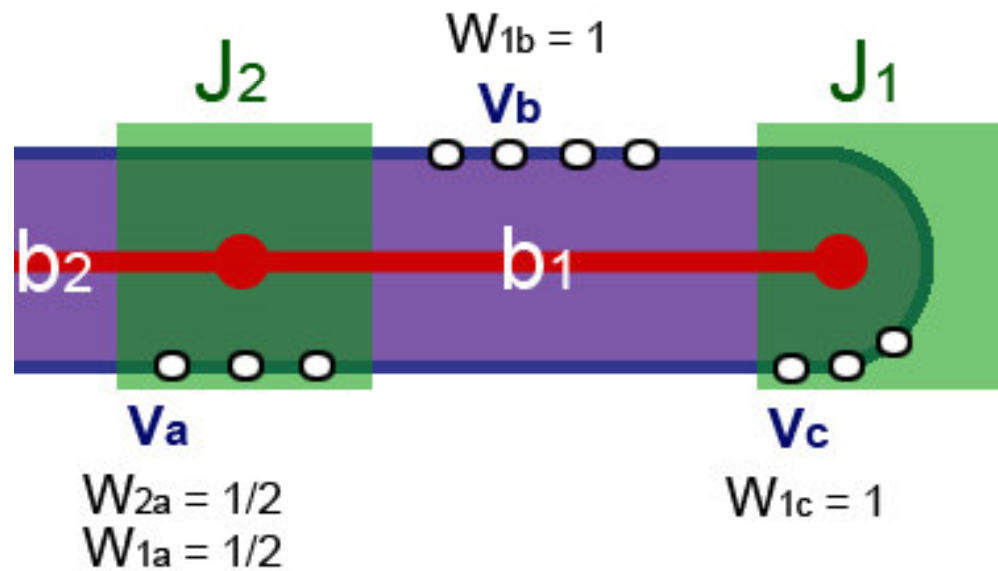
# System

## flowchart



# System

- Initialize  $W$



# System

- V-step
  - General constraints
    - Laplacian constraint
    - Position constraint
  - High level constraints
    - Length
    - Rigidity
    - Joint angle limit

# System

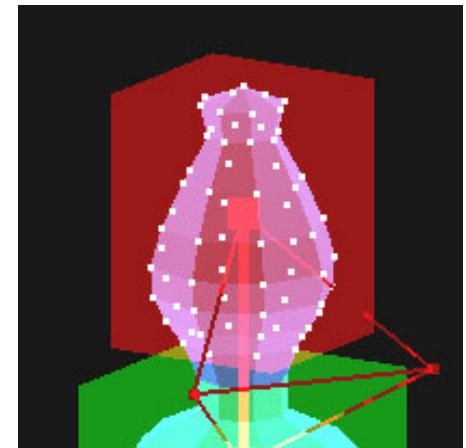
V-step

- **Laplacian constraint**
  - Preserve the detail of the surface

$$\left\| \mathbf{LX} - \frac{\mathbf{LX}'}{\|\mathbf{LX}'\|} \|\mathbf{L}\bar{\mathbf{X}}\| \right\|^2$$

- **Position constraint**
  - Allow direct manipulation of the mesh for intuitive design

$$\|\mathbf{PX} - \mathbf{X}'\|^2$$



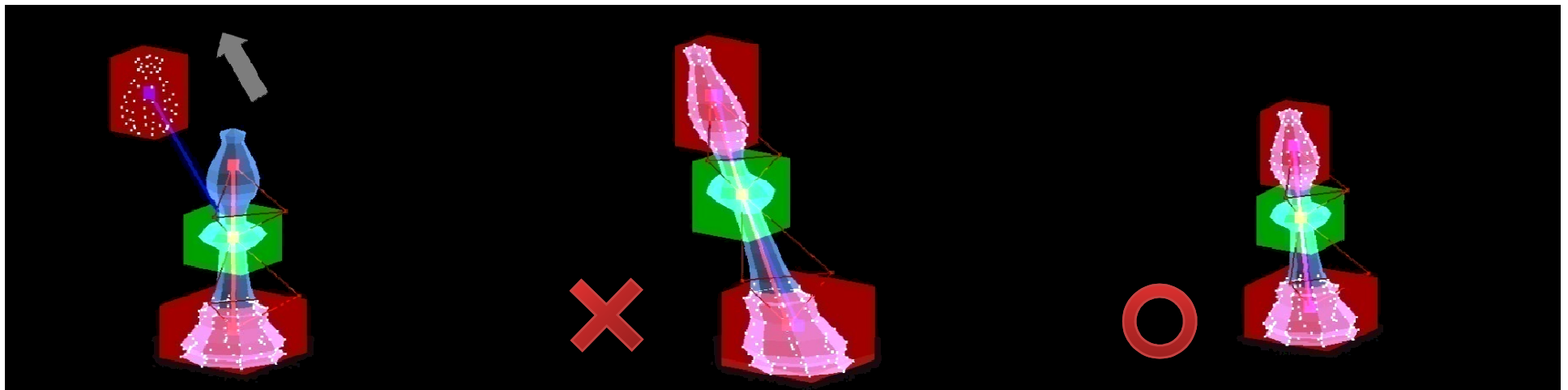
# System

V-step

- **Length constraint** [Zhou et al. 2007]
  - Control the length of the “bones”

$$\sum_{(i,j) \in \text{bones}} \left( \|\mathbf{v}_i - \mathbf{v}_j\| - L_{ij} \right)^2$$

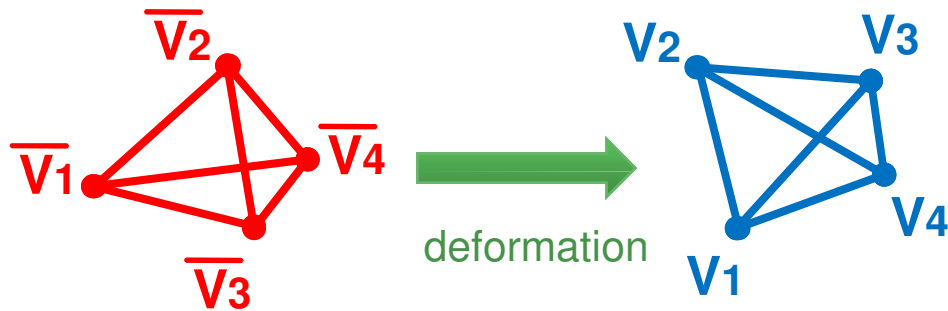
Original length of bone



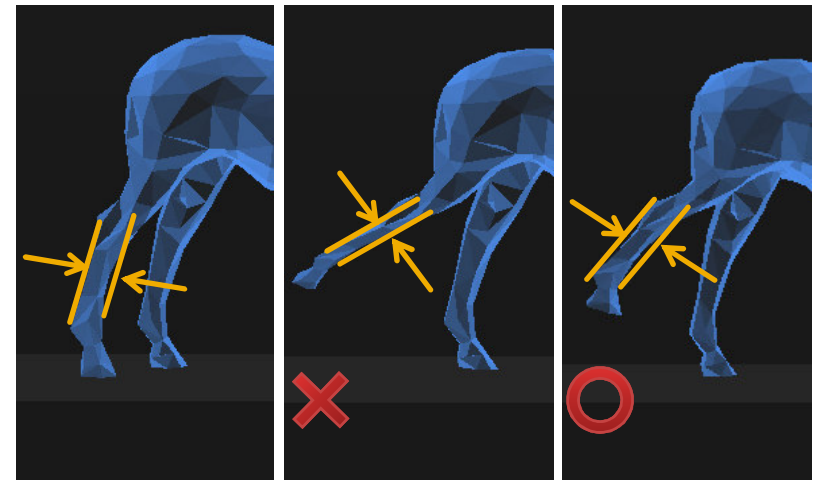
# System

## V-step

- **Rigidity constraint** [Zhou et al. 2007]
  - Force near-rigid deformation of skin around bones



$$\sum_{(i,j) \in tetra(b)} (\| \mathbf{v}_i - \mathbf{v}_j \| - l_{ij})^2$$

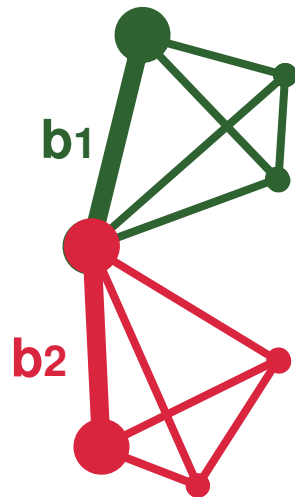


- $\mathbf{v}_i, \mathbf{v}_j$ : the position of tetravertices  $i, j$
- $l_{ij}$ : the distance between tetravertices  $i, j$

# System

V-step

- **Joint angle limit constraint** [Zhou et al. 2007]
  - Restrict the range of joint angles for added realism



$$\sum_{(i,j) \in \text{pairs}(b_1, b_2)} \|(\mathbf{v}_i - \mathbf{v}_j) - \theta_{ij}\|^2$$

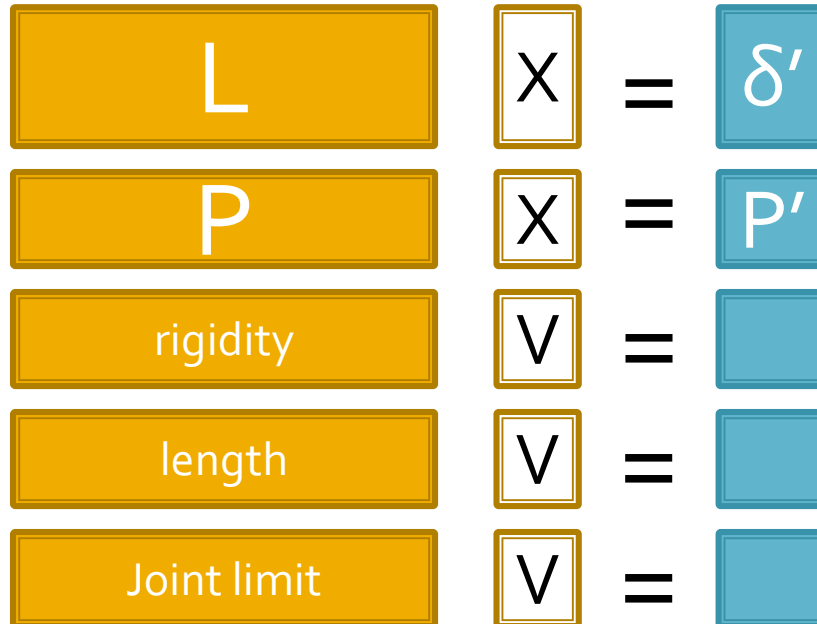
- $\mathbf{v}_i, \mathbf{v}_j$ : the position of tetravertices  $i, j$
- $\theta_{ij}$ : the target vector between tetravertices  $i, j$



# System

V-step

- Combine all constraints



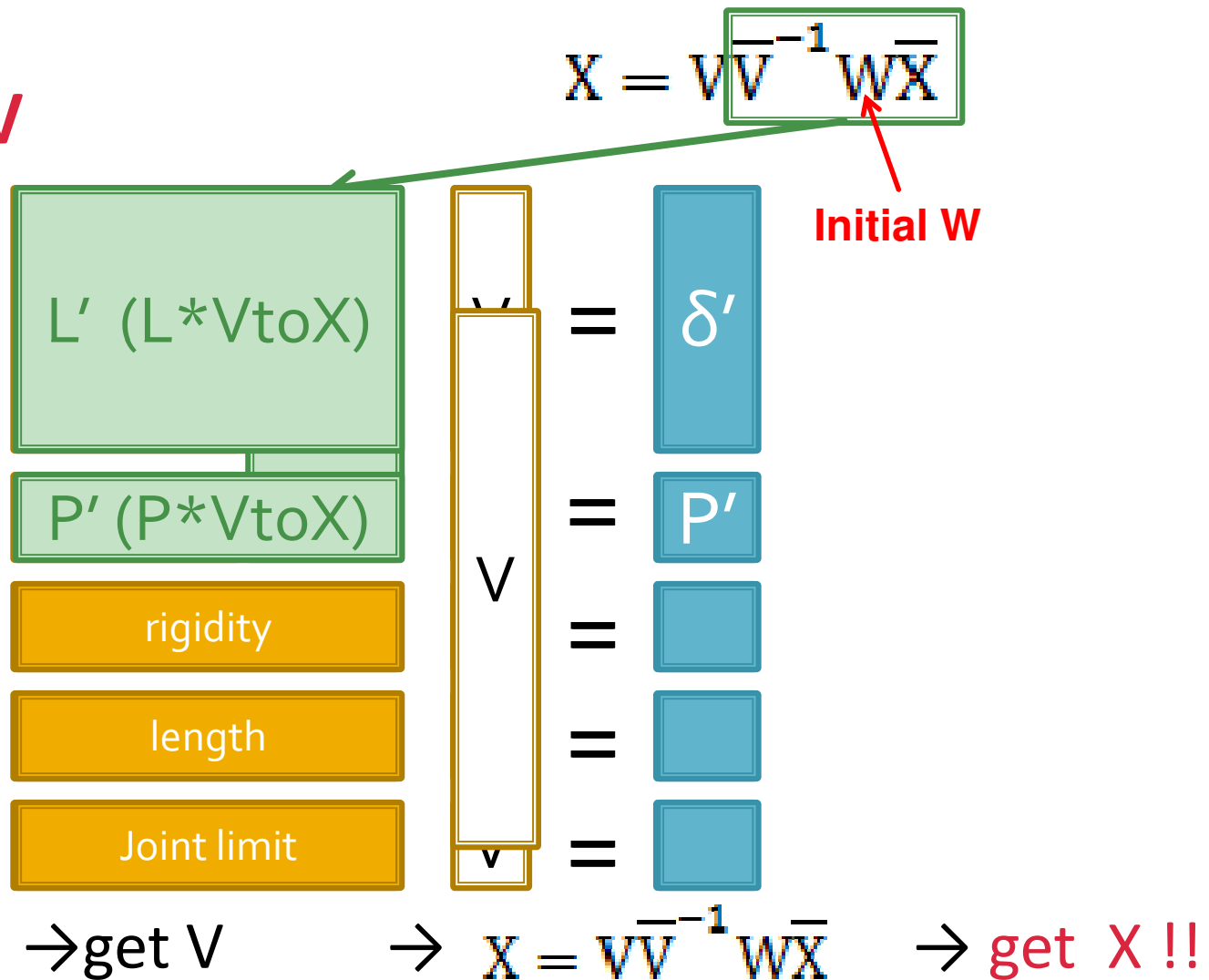
- V-step : Optimization of V
  - Method 1 : Only solve V then get X
  - Method 2 : Solve V and X at the same time



# System

V-step

Method 1 :  
Only solve V

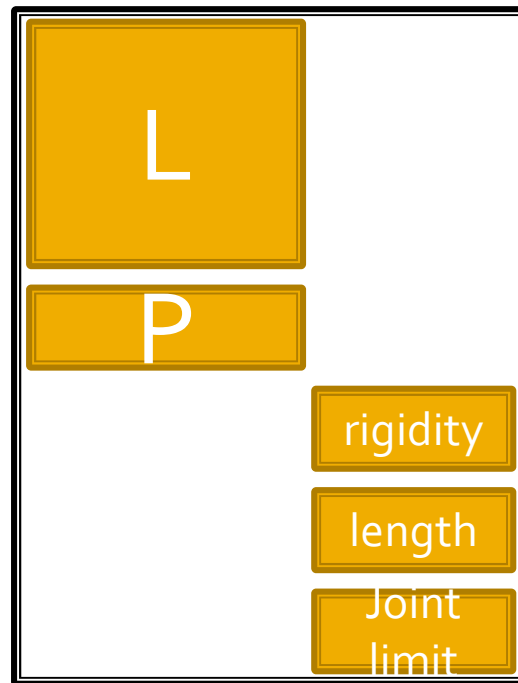


# System

V-step

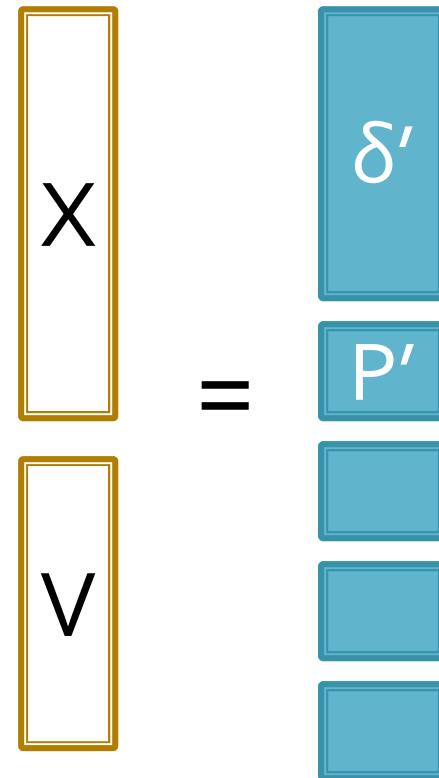
## Method 2 : Solve V & X

$$X = V V^{-1} W X$$



X and V relationship

→ get (X & V)



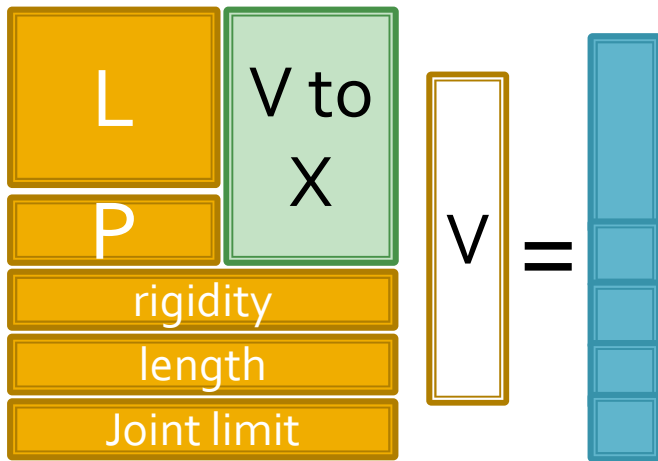
→ get X !!

# System

V-step

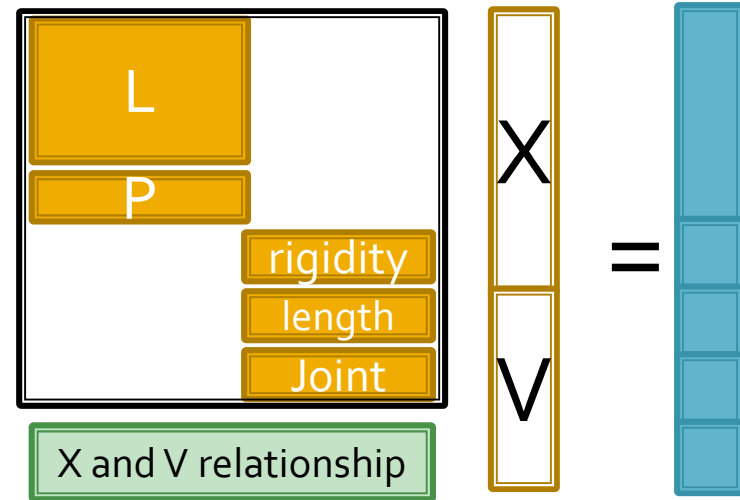
## Method 1

$2*nB + nJ$



## Method 2

$X.size + 2*nB + nJ$



$$V = (A^T A)^{-1} A^T b$$

$(2*nB + nJ) * (2*nB + nJ)$

**small**

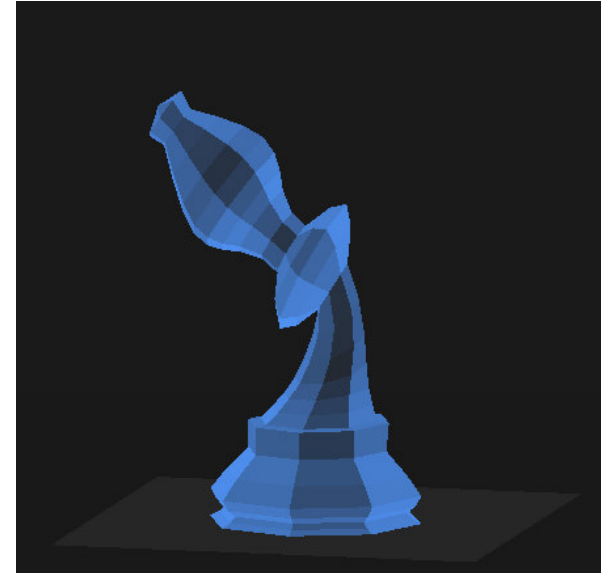
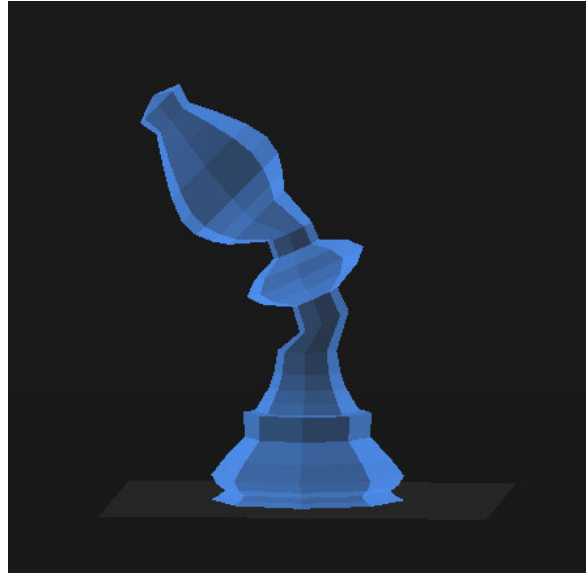
$(X.size + 2*nB + nJ) * (X.size + 2*nB + nJ)$

**large**

# System

V-step

Compare



	<b>Method 1</b>	<b>Method 2</b>
Dimension	Small	Large
Result	Worse	Better
Cost time	0.6 sec	1.2 sec (more)
W-step	necessary	optional
Large model	efficient	slower

# System

## W-step

### ■ W-step

- Constraints on Vertex Weights only
  - Smooth constraint

$$\varepsilon = \| LX - LX' \|^2 + \| PX - X' \|^2 \quad (\text{Laplacian \& Position constraint})$$

$$+ \sum_{(i,j) \in \text{pairs}(b_i, b_j)} \left( W_{bi} - \frac{1}{|N(i)|} \sum_{j \in N_i} W_{bj} \right)^2 \quad (\text{Smooth term})$$

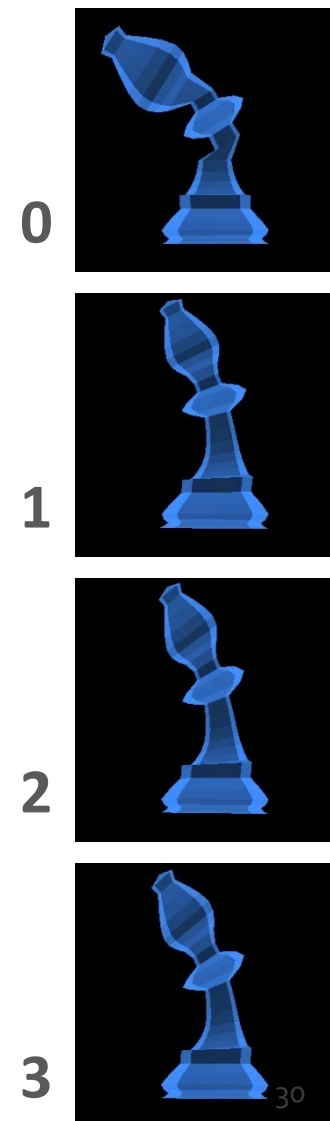
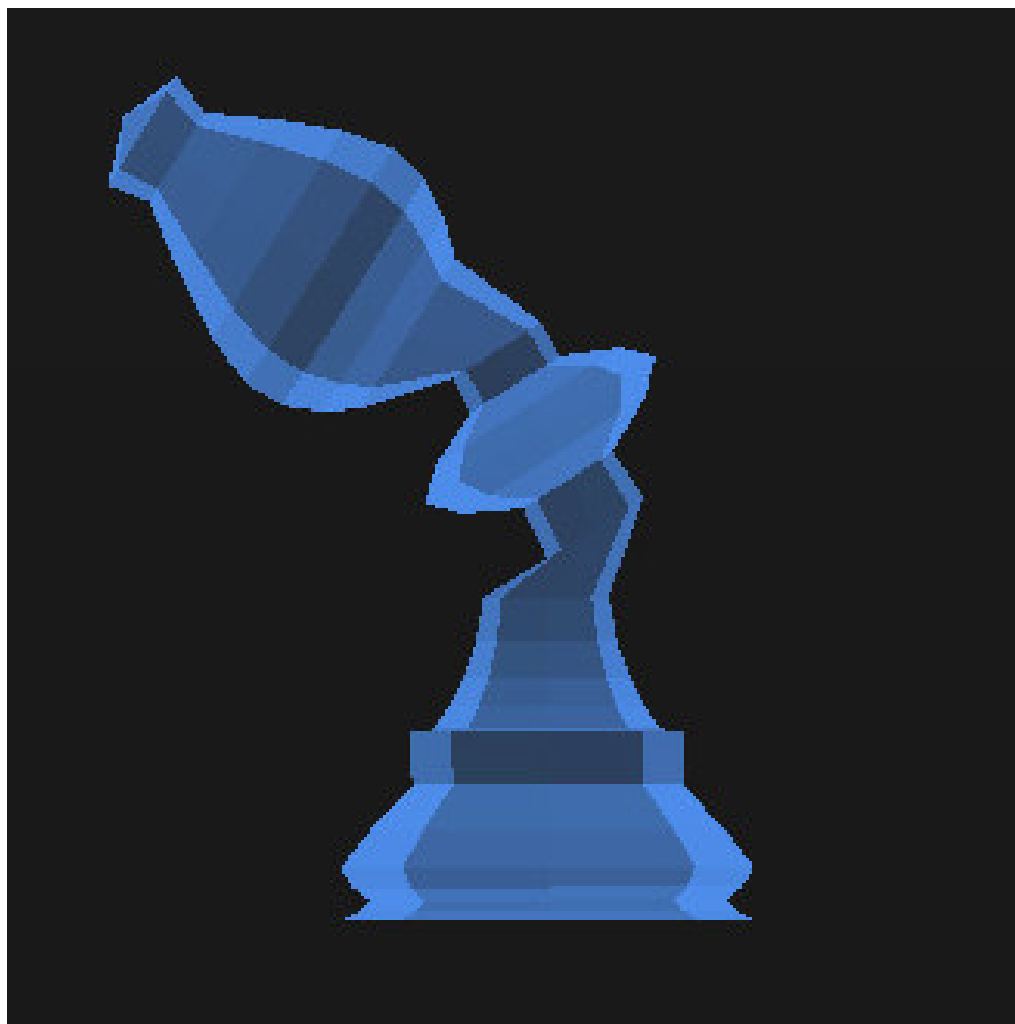
$$+ \sum_{i \in [1..n]} \left( \sum_{b \in B} W_{bj} - 1 \right)^2 \quad (\text{Normalization term})$$

# System

W-step

W-step  
times

0



# Outline

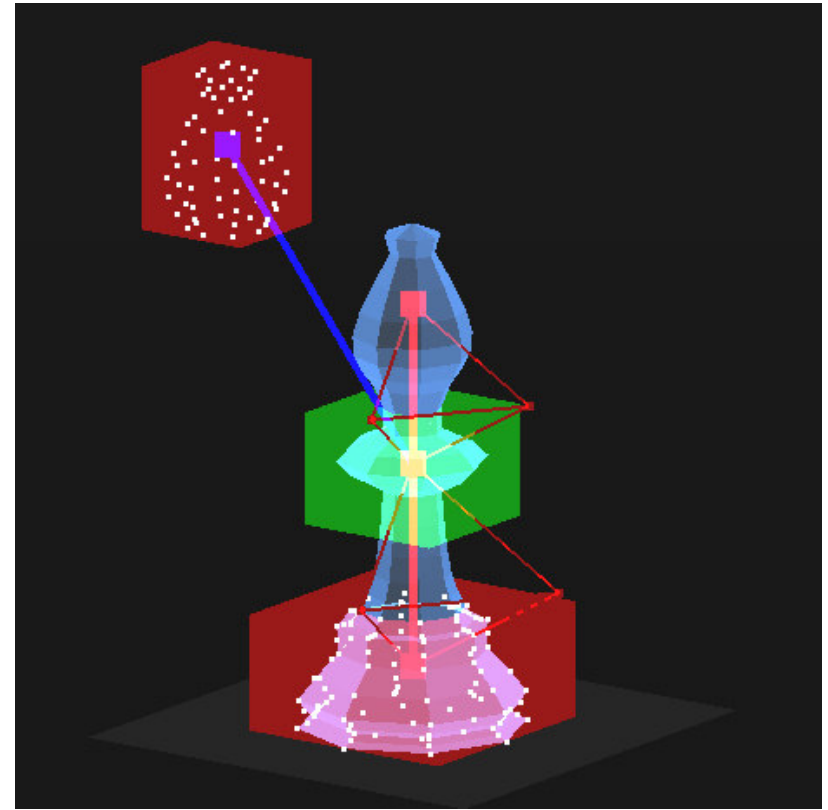
- Introduction
- Related Work
- System
- **Results**
- Conclusions and Future Work

# Results

## Length constraint



**Without** Length constraint

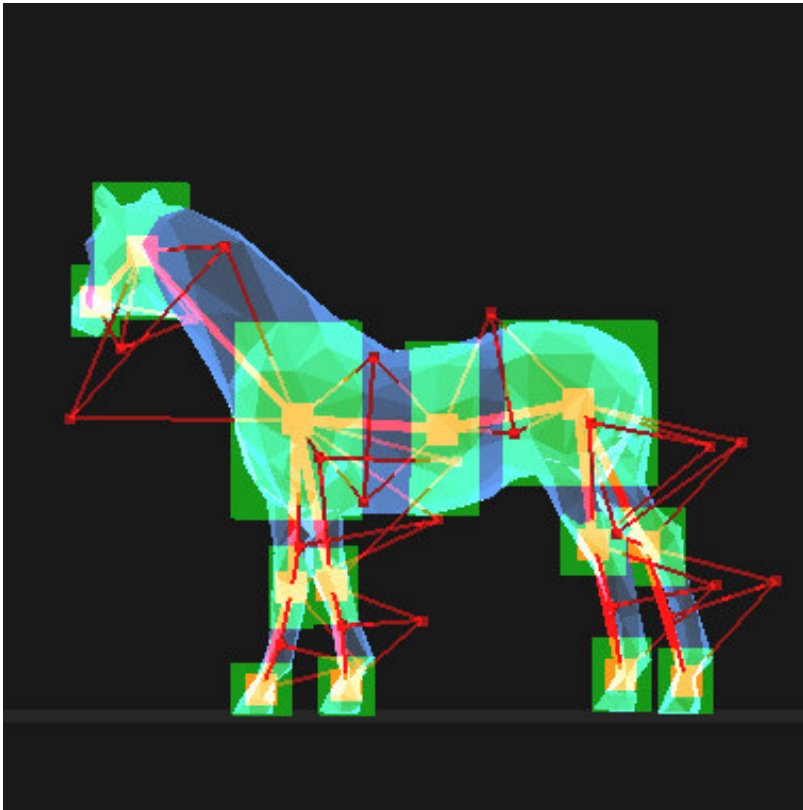


**With** Length constraint

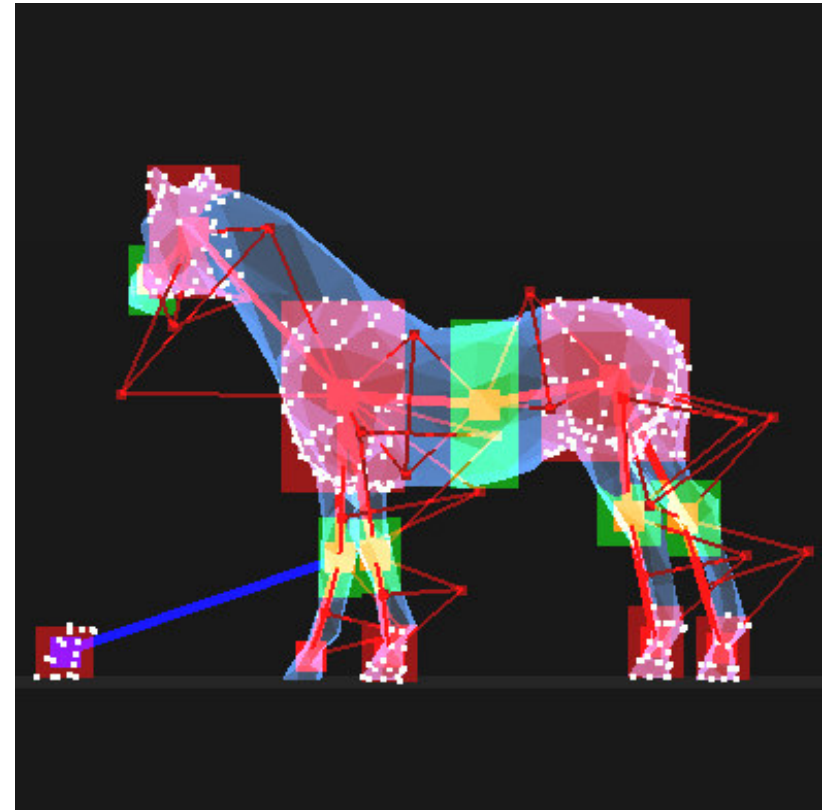


# Results

## Length constraint



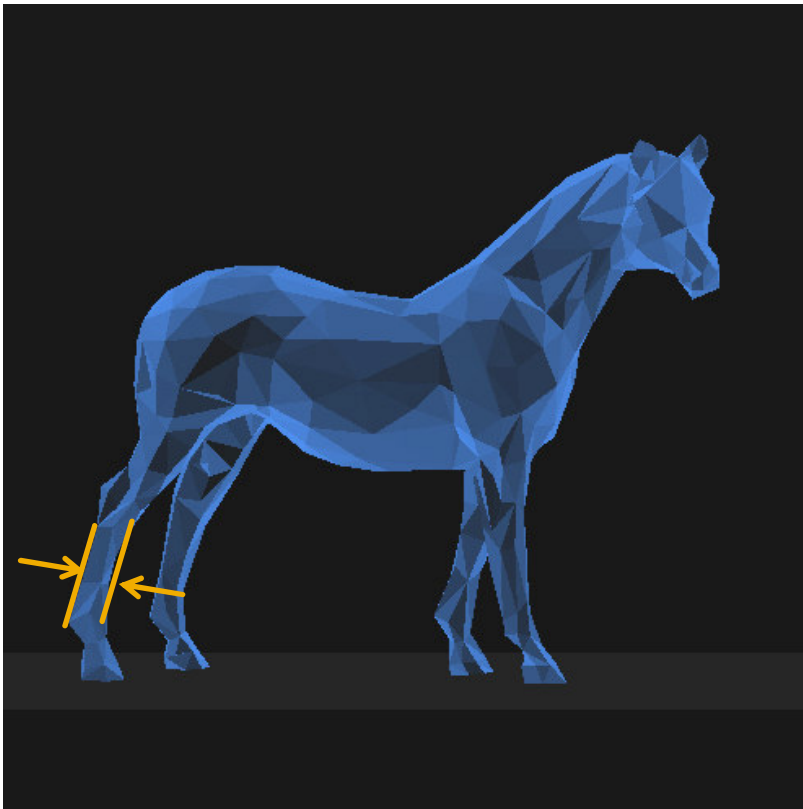
Without Length constraint



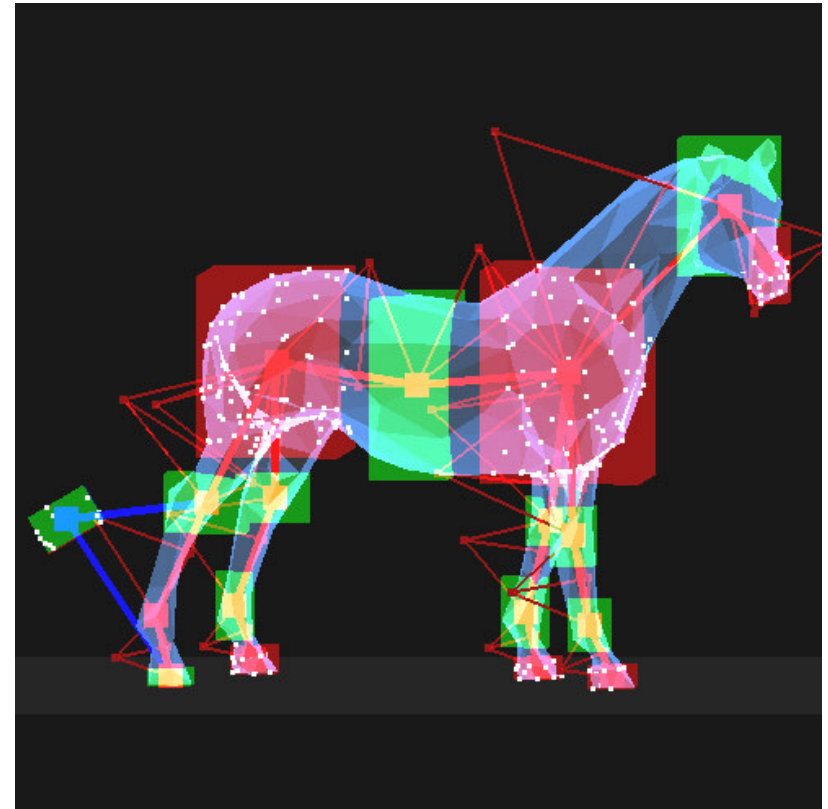
With Length constraint

# Results

## Rigidity constraint



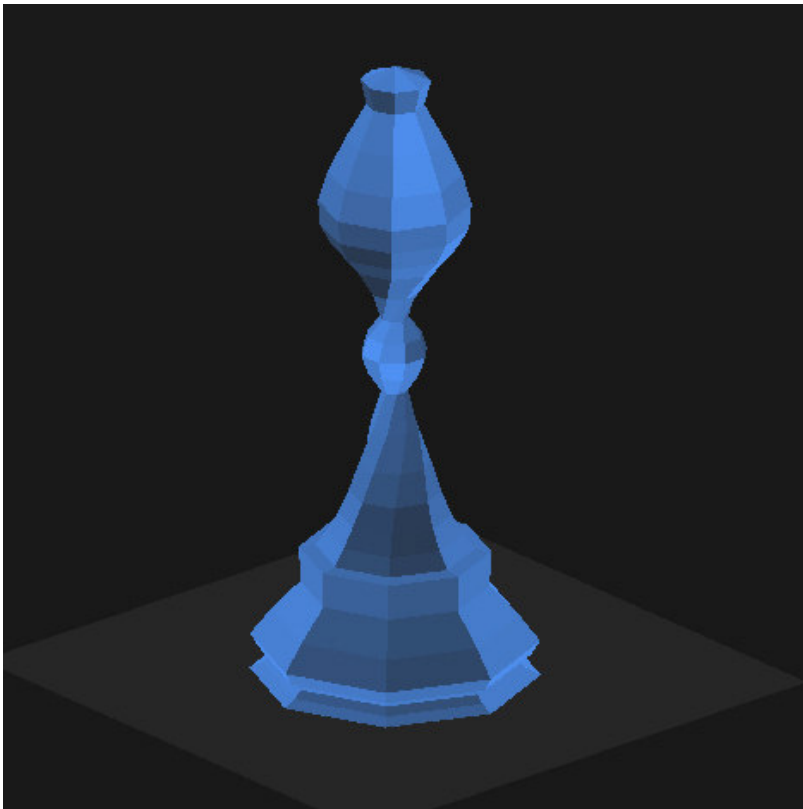
Without Rigidity constraint



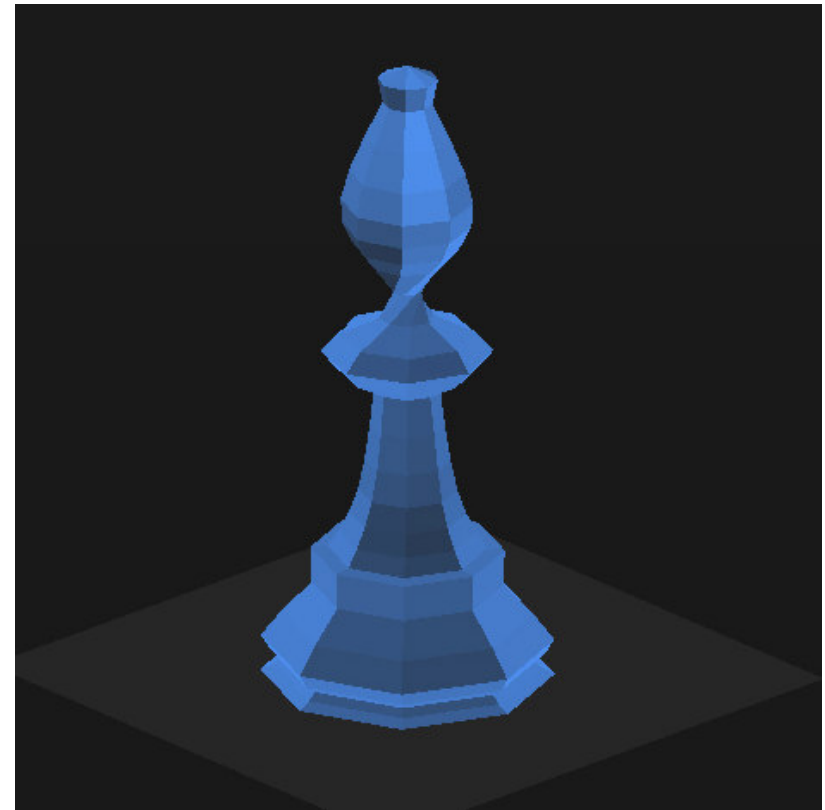
With Rigidity constraint

# Results

## Joint angle constraint



**Without** Joint angle constraint



**With** Joint angle constraint

# Results

## Some Interesting Results



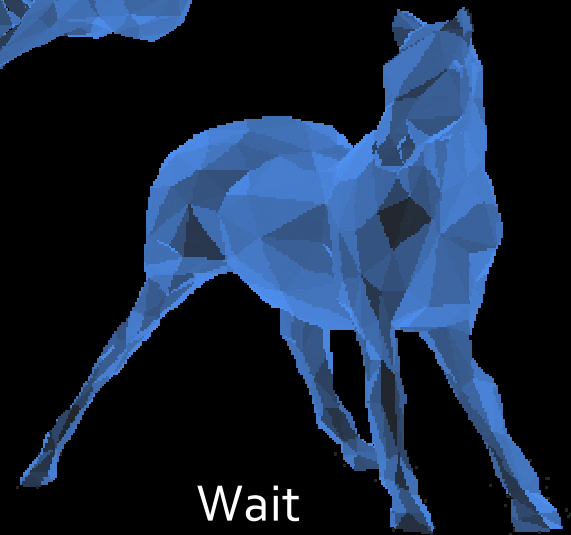
Stand



Sit



Jump



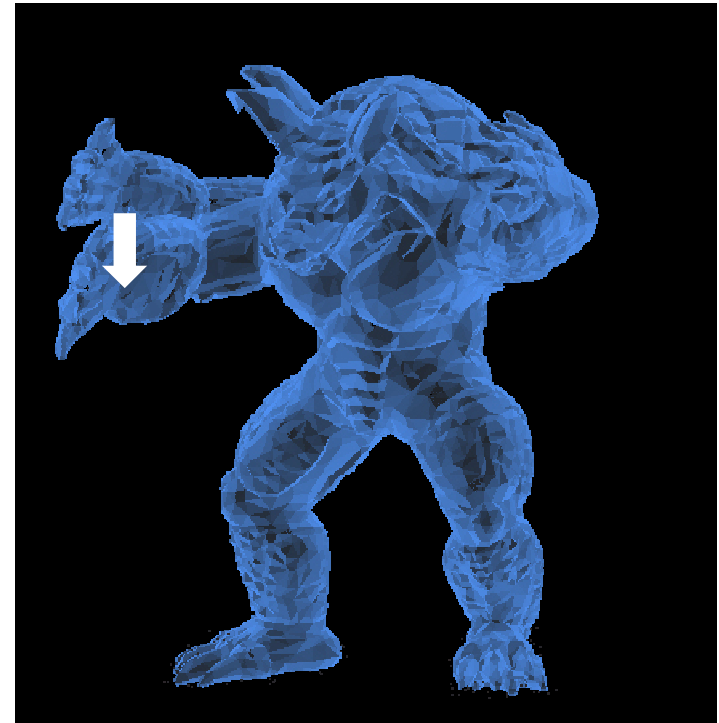
Wait

# Results

## Some Interesting Results



Raised up  
Dinosaur



Armadillo

# Outline

- Introduction
- Related Work
- System
- Results
- **Conclusions and Future Work**

# Conclusions and Future Work

- Conclusions
  - Convenience of manipulation on rigging and deformation
  - High-level constraints, and more natural and realistic deformed mesh
- Potential of the system
  - An interactive deformation platform
  - Various applications
    - Deformation transfer
    - Motion retargeting

# Conclusions and Future Work

- Future Work
  - Balance constraint
    - Mesh Puppetry [*Zhou et al. 2007*]
  - Auto-skeleton extraction
    - Domain Connected Graph: the Skeleton of a Closed 3D Shape for Animation [*Wu et al. 2006*]
  - Implement on multi-core processor





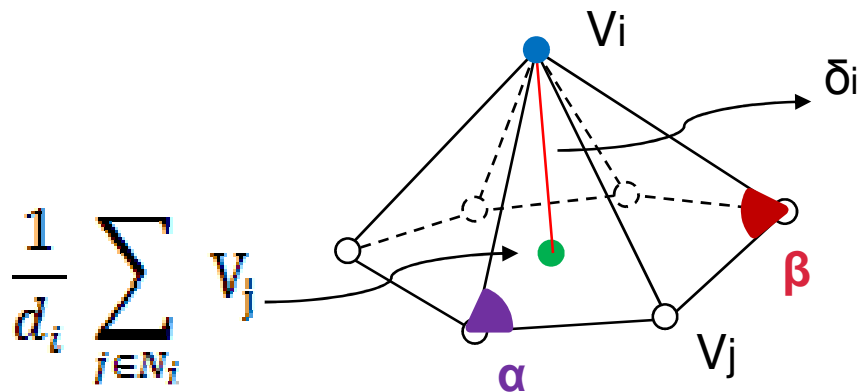
# Demo film

**Thank you**

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# Laplacain coordinates

- Laplacain coordinates



$$\delta_i = V_i - \frac{1}{d_i} \sum_{j \in N_i} V_j$$

- $d_i$  = degree of  $V_i$  (uniform weights)
- $d_i = \cot \alpha + \cot \beta$  (cotangent weights)

# Laplacain coordinates

- If we consider the rotation ...

$$\begin{array}{|c|} \hline V_{i.x} \quad V_{i.y} \quad V_{i.z} \quad 1 \\ \hline V_{1.x} \quad V_{1.y} \quad V_{1.z} \quad 1 \\ \hline V_{2.x} \quad V_{2.y} \quad V_{2.z} \quad 1 \\ \hline \vdots \\ \hline V_{n.x} \quad V_{n.y} \quad V_{n.z} \quad 1 \\ \hline \end{array} \quad \begin{array}{|c|} \hline \text{Rotation} \\ \hline \text{matrix} \\ \hline \end{array} \quad = \quad \begin{array}{|c|} \hline V_{i'.x} \quad V_{i'.y} \quad V_{i'.z} \quad 1 \\ \hline V_{1'.x} \quad V_{1'.y} \quad V_{1'.z} \quad 1 \\ \hline V_{2'.x} \quad V_{2'.y} \quad V_{2'.z} \quad 1 \\ \hline \vdots \\ \hline V_{n'.x} \quad V_{n'.y} \quad V_{n'.z} \quad 1 \\ \hline \end{array}$$

$$\mathbf{V} \quad \mathbf{R} \quad = \quad \mathbf{V}'$$

$$\mathbf{R} = (\mathbf{V}^T \mathbf{V})^{-1} \mathbf{V}^T \mathbf{V}'$$

# Laplacain coordinates

- If we consider the rotation ...

$$L V' = L V R$$

$$V R = V'$$

$$= L V (V^T V)^{-1} V^T V'$$

$$R = (V^T V)^{-1} V^T V'$$

$$L (V' - V (V^T V)^{-1} V^T V') = 0$$

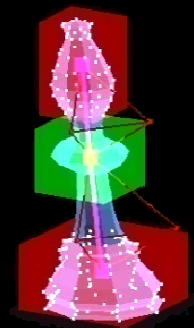
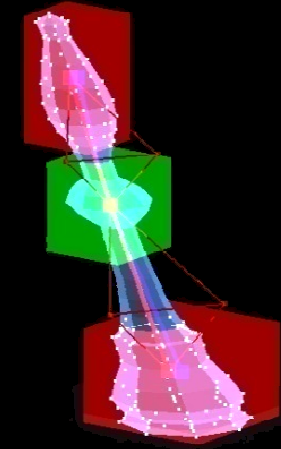
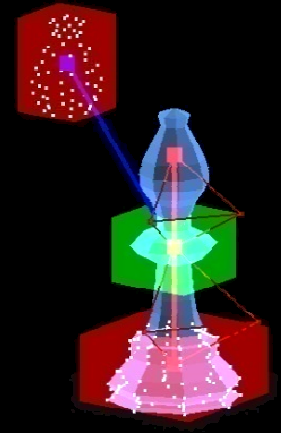
$$L (1 - V (V^T V)^{-1} V^T) V' = 0$$

# Length constraint

- Length constraint
  - Control the length of the “bones”

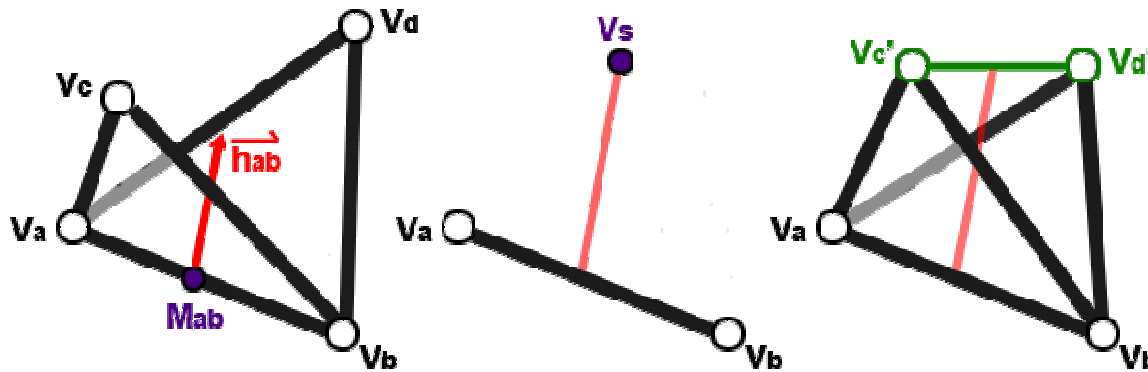
$$\sum_{(i,j) \in \text{bones}} (\|\mathbf{v}_i - \mathbf{v}_j\| - L_{ij})^2$$

$$\left\| (\mathbf{v}_i - \mathbf{v}_j) - \frac{\mathbf{v}_i' - \mathbf{v}_j'}{\|\mathbf{v}_i' - \mathbf{v}_j'\|} L_{ij} \right\|^2$$



# After Rigidity constraint

- **Rigidity constraint**
  - After deformation, we rebuild new tetrahedrons to be reused in next times

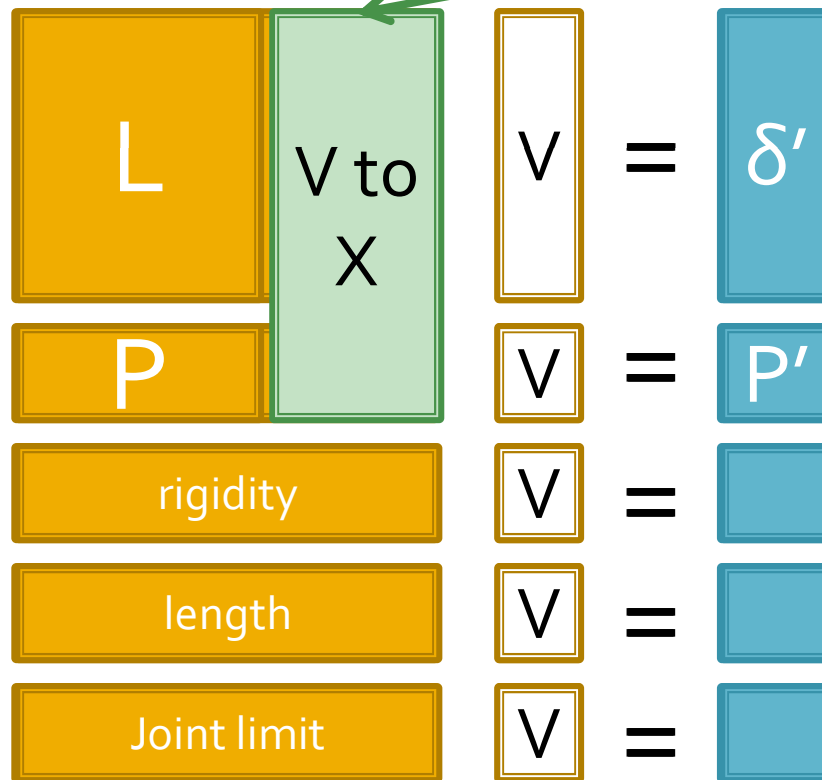


# System

Method 1 :  
Only solve V

$$X = V \bar{V}^{-1} W X$$

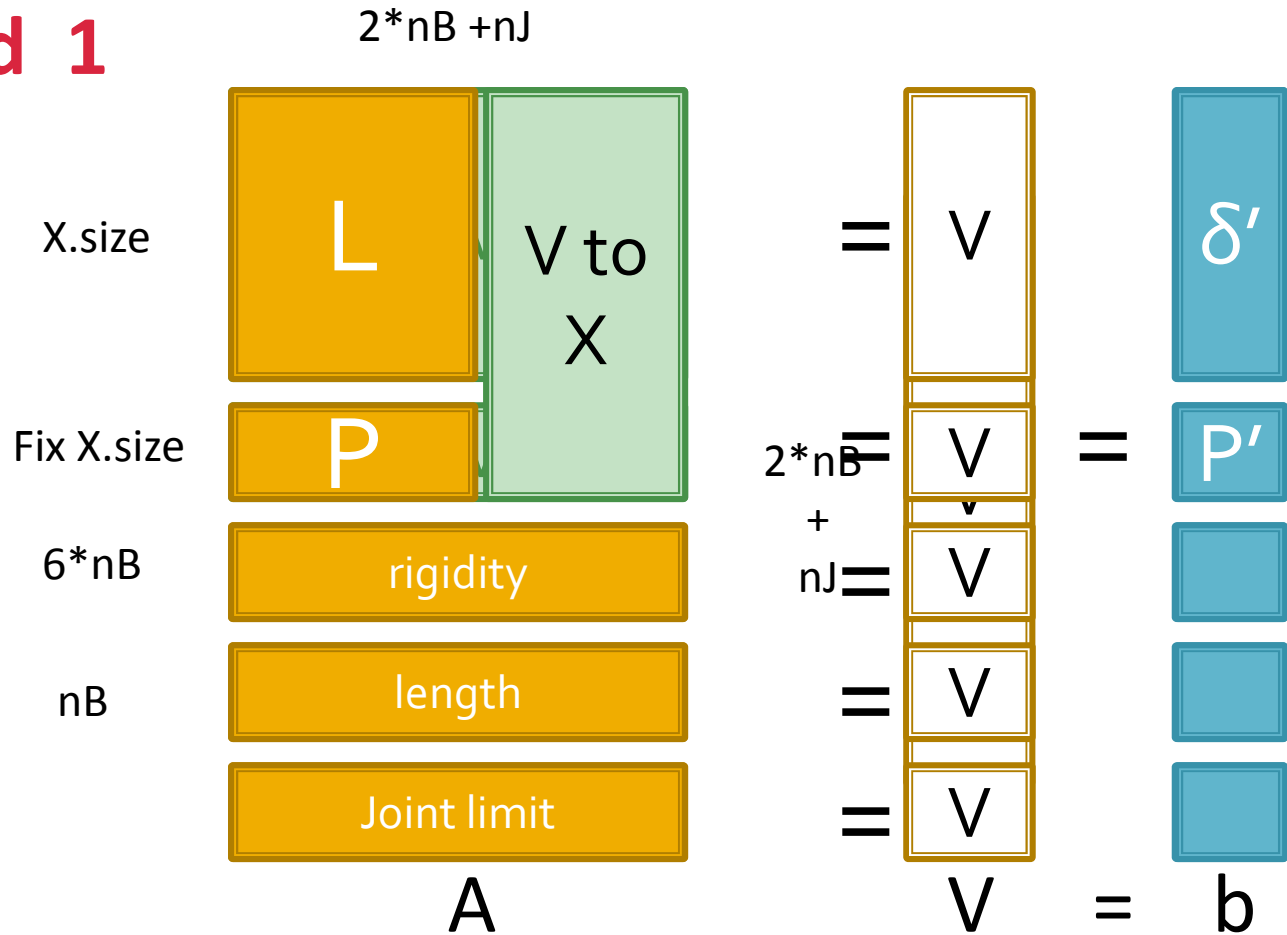
Initial W





# System

## Method 1



→ get  $V$

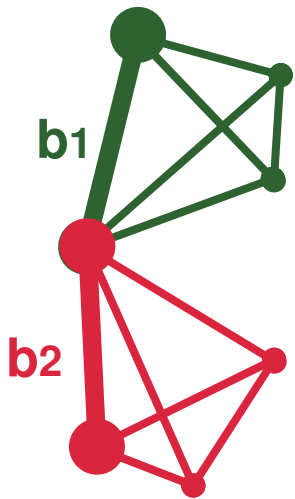
$$\rightarrow X = V V^{-1} W X$$

→ get  $X !!$

# Joint angle limit constraint

V-step

- Joint angle limit constraint
  - Restrict the range of joint angles for added realism



$$\sum_{(i,j) \in \text{pairs}(b_1, b_2)} \left\| (\mathbf{v}_i - \mathbf{v}_j) - \theta_{ij} \right\|^2$$



$$\sum_{(i,j) \in \text{pairs}(b_i, b_j)} \left( \|\mathbf{v}_i - \mathbf{v}_j\| - d_{ikj} l_{ikj} \right)^2$$

$$l_{ikj} = \sqrt{l_{ik}^2 + l_{jk}^2 - 2 l_{ik} l_{jk} \cos \theta_{ikj}}$$

- $\mathbf{v}_i, \mathbf{v}_j$ : the position of tetravertices  $i, j$
- $\theta_{ij}$ : the target vector between tetravertices  $i, j$

